

# **“ECONOMIES OF SCALE AND SCOPE IN ONLINE EDUCATION”**

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## **ABSTRACT:**

This paper seeks to understand if online education is cost effective relative to the cost structure of American universities. Using data from the Integrated Postsecondary Education Data System, universities are observed as having five primary outputs: undergraduate enrollment, graduate enrollment, research output, medical schools, and online enrollment. Transcendental logarithmic cost functions are estimated to produce cost curves, given these five outputs. Using mean data from the descriptive statistics as well as the coefficient values from the cost functions, ray economies of scale and economies of scope are estimated. Results suggest that economies of scale exist for both public and private universities. Moreover, economies of scope exist for both public and private universities with online education output.

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## II. INTRODUCTION

The cost of higher education, whether public or private, continues to drastically increase. According to The College Board, the price of education has increased ten-fold since 1970, far outpacing the rate of inflation (Ma et al, 2015) The price of tuition, room and board for a public university has increased from \$1,405 in 1970 to \$19,548 in 2015, while private universities have increased from \$2,929 to \$43,921 (Ma et al, 2015).

Furthermore, according to The College Board, “Between 2005-06 and 2015-16, published in-state tuition and fees at public four-year institutions increased at an average rate of 3.4% per year beyond inflation, compared to average annual rates of increase of 4.2% between 1985-86 and 1995-96 and 4.3% between 1995-96 and 2005-06” (Ma et al, 2015). Thus as the cost of education continues to increase, the composition and structure of higher learning changes with it.

Distance learning is increasingly becoming an integral part of higher education. As technology proliferates, the role distance learning plays in higher education continues to expand. As of 2014, over two-thirds of academic leaders consider online education critical to the long-term success of their institution (Allen et al, 2016). And the amalgamation of online education with higher education continues to grow. Over one in four students (28%) took at least one distance education course in 2014, a 3.9% year-to-year increase from 2013 (Allen et al, 2016). Even as enrollment in higher education decreases, the amount of online education increases. And this increase may come with pleasant side effects.

McPherson and Bacow (2015) state that “larger classrooms, less student-faculty contact, and less intensive hands on learning” are ways to decrease the costs of education.

These factors may all occur through distance learning. Since the constraint of physical space is not binding in online classes, class sizes can increase drastically. Because classes are held primarily online, less student-faculty (face-to-face) contact and hands on learning occur. Yet this cost saving, if it exists, has been minute for students, much smaller than the theorized savings from online education (Deming et al, 2015).

While the decrease in the costs of education is certainly plausible, negative outcomes on the quality of education may also occur. The creation of a disconnect between faculty and students as well as decreased face-to-face interaction may indeed lead to a reduction in the quality of education. The decline in effectiveness of instruction via distance learning may outweigh the cost-savings involved. Thorough and well-designed studies to measure the effectiveness of online learning are inexistent while those that do exist find statistically little to no differences in student performance (McPherson & Bacow, 2015). McPherson and Bacow postulate that the problem lies in the lack of an accurate measure for effectiveness. Because of the lack of an accurate measure, the effects on the quality of education cannot be measured and as such, are beyond the scope of this paper.

Thus, this paper seeks to understand if online education is cost effective relative to the cost structure of American universities. Specifically, when online education is considered an output of a university, does the average cost of said university producing a bundle of goods decrease when the output is expanded? Moreover, is it cost effective to produce online education in conjunction with the other goods of a university, or should another institution produce online education separately? Using data from the Integrated Post-Secondary Education Database System, I will expand on previous literature that

modeled the cost function of universities and integrate online enrollment as another output of higher education. Moreover, I will calculate economies of scope and scale to answer whether online education should be produced in conjunction with other goods and if the average cost of a university decreases when output is expanded. After, I will use my results and analysis for policy recommendations for the future.

### **III. LITERATURE REVIEW**

Cohn, Rhine, and Santos (1989) utilize the theory of multi-product firms to estimate the cost functions for universities. Cohn et al (1989) sought to estimate the degree of economies of scale and whether there are economies of scope between teaching and research. Using a fixed cost quadratic function, Cohn et. al measure teaching output by full time equivalent students and research output by research expenditure. Data for their analysis came from a cross-sectional survey by the Higher Educational General Information Survey for the 1981-82 academic year. Results show that although the cost functions for private and public universities are statistically structured differently, both the public and private sectors benefit from economies of scale as output rises. Cohn et al also found that complex institutions of higher education, those that focus on undergraduate and graduate instruction as well as research, are less costly than IHEs that focus on only one output.

Expanding Cohn et al (1989)'s model of a multiproduct cost function, Laband and Lentz (2005) looked to find the impact of extension services on higher education, and to what extent are extension services characterized by economies and diseconomies of scale and scope. Using a flexible fixed cost quadratic function, Laband and Lentz measure teaching output with full time enrollment, measure research output by the number of

research publications, and they include a measure of extension services as the number of in-person extension contacts with constituents. Data for the analysis came from the National Center for Education Statistics 1995-96 fiscal year. Results show that extension service providing universities have economies of scale, yet no economies or diseconomies of scope between extension services and undergraduate education, graduate education, or research was found.

De Groot, McMahon, and Volkwein (1991) focused on doctorate-granting universities with a large research emphasis. Using a translog cost function, teaching output is measured by full time enrollment for graduate and undergraduate students and research output measured by the number of research publications. Data used came from multiple sources. Cost data was obtained for the 1983 fiscal year from the National Center of Educational Statistics and research output data from the study of the Council of Associated Research Councils. Results found economies of scale for the average institution in teaching and research output and economies of scope between undergraduate and graduate instruction. However, unlike previous research, they did not find economies of scope between graduate instruction and research.

Johnes (1996) analyzes United Kingdom universities to evaluate ray economies of scale (a measure of economies of scale for multi-product firms, keeping output composition constant), product-specific economies of scale, and economies of scope. Using a quadratic multi-product cost function, Johnes measure teaching output by full-time equivalent undergraduates and postgraduates and research output by the measure of research activity. Using OLS regression, results show product-specific scale economies for graduate enrollment and research output, and economies of scope for teaching and

research output.

Deming, Goldin, Katz and Yuchtman (2015) analyze the effects of online education on tuition. Specifically, Deming et al studied whether online education decreases the cost curve in higher education at open access and less-selective postsecondary institutions. Data used was obtained from the Integrated Postsecondary Education Data System of US Department of Education. Deming et al deviate from past research and focus on tuition, the price charged to students, instead of the internal cost structure of the university. However, costs and tuition are intrinsically linked, as the costs of a university will later affect the tuition charged. In their research, they regress tuition and fees, instead of costs, on the share of students enrolled in online education, while controlling for sector, urbanity, and measures of selectivity. Results find that a 10 percent increase in the share of students taking all online courses at public universities decreases prices by 1.4 percent. However, they found no impact of online classes on the price of private institutions.

#### **IV. METHODOLOGY**

In order to accurately depict the cost structure of higher education, the cost function must be constructed. General cost theory states that a firm seeks to minimize the cost of producing a bundle of goods, given the restraint of the optimal amount of goods to produce. The respective cost function then is derived from the production function. However, unless the specific production function is known *ex ante*, one cannot derive the functional form of the cost function. Thus, we need some flexibility when estimating said function. Following from the methodology of DeGroot et al (1991), I estimate the

function using the transcendental logarithmic cost function for multi-product firms, an approximation of a second-order Taylor series to an arbitrary cost function:

$$\log C(q_1, q_2, q_3, q_4) = a_0 + \sum_i a_i \log(q_i) + \sum_{i \leq j} a_j \log(q_i) \log(q_j) + \sum_k a_k \log(p_i) + \sum_{i \leq j} a_l \log(p_i) \log(q_j)$$

in which

C = total costs

q<sub>1</sub> = undergraduate output

q<sub>2</sub> = graduate output

q<sub>3</sub> = research output

q<sub>4</sub> = online learning outputs

p<sub>1</sub> = average instructor salary

a<sub>0</sub> = constant

a<sub>i</sub>, a<sub>j</sub>, a<sub>k</sub>, a<sub>l</sub> = coefficients.

Moreover, in addition to the independent variables included for the output, a dummy variable is incorporated for the existence of a medical school at the university. Because of previous research on structural differences (Cohn et al, 1989), the data is separated into private institutions and public institutions to account for cost dissimilarities.

### *Economies of Scale and Scope*

I follow the standard methodology of calculating economies of scale and scope first presented by Baumol, Panzar and Willig (1982) and continued by Cohn et al (1989), DeGroot et al (1991), and Laband and Lentz (2005). Ray economies of scale (a measure of economies of scale for multi-product firms) are analyzed to measure if the average costs of producing a university's outputs increase or decrease when output is expanded. Furthermore, economies of scope are also analyzed to measure if a product is produced cheaper in conjunction with another product instead of both separately.

1. *Ray Economies of Scale*: Assume a multi-product firm can produce  $n$  different types of outputs. Let  $C(Y)$  equal the cost of producing  $Y = \sum_{i=1}^n Y_i$  outputs. Let  $C(Y_i)$  be the marginal cost of producing only the  $Y_i$  outputs, such that:

$$(1) \quad C(Y_i) = \frac{\partial C(Y)}{\partial Y_i}$$

Then ray economies of scale,  $R(Y)$ , are calculated by:

$$(2) \quad R(Y) = \frac{C(Y)}{\sum_i Y_i C(Y_i)}$$

Ray economies of scale exist when  $R(Y)$  is greater than one. Conversely, Ray diseconomies of scale exist when  $R(Y)$  is less than one.

2. *Economies of Scope*: Let  $C(Y_m)$  equal the cost of producing only product  $m$ . Thus  $C(Y_{n-m})$  is equal to the cost of producing all  $n$  products except  $m$ . Then, economies of scope,  $SC_m(Y)$  can be computed by:

$$(3) \quad SC_m(Y) = \frac{C(Y_m) + C(Y_{n-m}) - C(Y)}{C(Y)}$$

where economies of scope exist when (3) is greater than zero.

## VI. DATA AND LIMITATIONS

Data for my analysis comes from the Integrated Postsecondary Education Data System, a database by The United States Department of Education. All data used comes from the 2014 academic year. Output for undergraduates and graduates is measured by fall enrollment. Research output is measured by research expenditures, a limitation of the study. Ideally, research output should be measured by the number of research publications an institution has in a year, yet current data on this is not available. As such, this study follows the proxy used in previous research of research expenditures (Cohn et al, 1989). Preferably both input prices for instruction (undergraduate and graduate) and

research output should be included. Yet lack of data for the latter restricts only the input price for instruction to be included. In order to derive the instruction price input, the total salary for instructors was divided by the amount of individuals employed as instructors to obtain the average instructor salary. Last, in order to derive the online education output the percent of students taking some online education courses or exclusively online courses was multiplied by the number of students enrolled at each university to acquire the number of undergraduate and graduate students taking online classes, respectively, and then also summed to garner the total number of students enrolled in online education courses. This number was subtracted from that of undergraduate and graduate enrollment to ensure double counting of students does not occur. A sample of 1,917 for public universities and 1,867 for private universities was used in the analysis, omitting any universities that had missing data. Table 1 lists the variables used as well as their descriptive statistics.

## **VII. RESULTS**

Regression analysis is separated into two scenarios. In scenario one, costs are specified as a function of five outputs: undergraduate enrollment, graduate enrollment, research, online enrollment, and medical school. Meanwhile, scenario two separates online enrollment into undergraduate and graduate online, consequently specifying a six output cost function: undergraduate enrollment, graduate enrollment, research, online undergraduate enrollment, online graduate enrollment, and medical school. Thus while scenario one will be used to measure the cost effectiveness of online learning as an aggregate, scenario two will be used to measure the cost effectiveness of undergraduate and graduate online learning separately. Tables 2-3 show the OLS- Regression results

using data for both public and private universities. With some exceptions, the coefficients on almost all variables are statistically significant. It is important to note that the coefficients vary significantly for private and public universities, suggesting indeed difference in cost structures. For example, the presence of a medical school for public universities increases the costs at a much faster rate than private universities. Furthermore, it is interesting to note that the coefficient on the interaction term between research and graduate enrollment is negative for both public and private universities in scenario two (suggesting dis-complementarity between the two), but positive in scenario four for public universities (suggesting complementarity).

#### **(i) ECONOMIES OF SCALE & SCOPE**

The fundamental purpose of the study lies in the calculation of ray economies of scale and economies of scope. Taking mean data from the descriptive statistics (Table 1) as well as the coefficient values from the OLS-Regression analysis (Tables 2-3), economies of scale and scope were calculated using the formulas specified in equations (1), (2), and (3). Results are presented in Table 3.

It should be noted that to correct for statistical bias in the log-transformed data, methods produced by Newman (1993) were used. For my analysis, the logarithmic transform of the regression model was back-transformed to the original power model. In doing so, the transform of the error term was omitted. To correct for the bias, we assume the regression residuals are normally distributed. Thus, when calculating economies of scale and scope, the following is included in the equation:

$$10^{\varepsilon} = 10^{MSE/2}$$

in which

$$\begin{aligned}\varepsilon &= \text{Error Term} \\ \text{MSE} &= \text{Mean Squared Error.}\end{aligned}$$

Using scenarios one and two at the mean level of output, ray economies of scale exist for public and private universities in both scenarios. This is in line with earlier findings that found both private and public universities to have ray economies of scale. Furthermore, at the mean level of output in scenario one, economies of scope exist for online output, undergraduate enrollment, and graduate enrollment in both private and public universities. However, differing from previous literature, the results indicate no economies of scope exist in research output for both public and private universities. At the mean level of output in scenario two, economies of scope exist for undergraduate and graduate enrollment as well as research output. Moreover, economies of scope, particularly in undergraduate enrollment are much higher than in scenario one. When separating online enrollment into undergraduate and graduate enrollment, economies of scope are found in online graduate enrollment for both public and private universities. Conversely, economies of scope are found in online undergraduate enrollment for public universities, but are absent for private universities.

Reasons for these discrepancies are multiple. Human error may exist in the calculation of the economies of scope and scale. Further review is needed to check the accuracy of the results. Moreover, using research expenditures (a less-wanted proxy) may skew the results since it may be directly correlated with total expenditures. Further analysis is needed to understand the limitations these issues may bring and what affect they have on the results.

## VIII. CONCLUSION

The main purpose of the study has been to estimate a multiple output cost function for private and public universities in order to study economies of scope and scale. After analysis of the results, it is found that ray economies of scale exist for both public and private universities, with public universities having stronger economies of scale. This has several policy implications. Though both have economies of scale, public universities will face lower average costs to expand at the mean level of output, while private universities face higher average costs if they want to expand. As such, private universities may consider remaining small in enrollment, research, and online learning output while public universities may consider expanding.

Additionally, economies of scope exist for online enrollment for both public and private universities. However, when splitting online enrollment into undergraduate and graduate enrollment, economies of scope exist in online graduate enrollment for both public and private universities. Yet economies of scope exist in online undergraduate enrollment for only public universities. Thus, public institutions will find that it costs less to produce online education in conjunction with enrollment and research, instead of producing the product separately, while private institutions will have decreased costs from producing only online graduate enrollment in combination with enrollment and research. Cognizant of the economies of scope evident in both public and private institutions, universities may find increased cost savings with the expansion of online and distance learning.

As stated earlier though, caution should be taken when reviewing the results. Possible correlation between variables (especially the variable of interest) may cause abnormal and inaccurate results. Further examination is needed to verify the results.

Looking forward, of central importance to the topic is the effect online education has on the quality and effectiveness of teaching. Future areas of research should focus on the change in quality, if it occurs, from increased online education as well as if the change in quality outweighs the cost-savings involved with expansion of online learning.

## IX. APPENDIX

**Table 1: Descriptive Statistics**

Variable Symbol	Description	Private Universities		Public Universities	
		Mean	Standard Deviations	Mean	Standard Deviation
MED_SCH	Medical School	0.043	0.205	0.052	0.222
UGRAD	Undergraduate Enrollment	1,276.8	2,280.4	5,069.1	6,416.8
GRAD	Graduate Enrollment	494.7	1,475.4	527.3	1,592.5
RES_TOTAL	Research Expenditures	10,575,766.8	77,595,191.1	16,897,965.2	79,660,295
UGRAD_ONL	Undergraduate Online Enrollment	328.3	2,095.3	1,809	2,690.4
GRAD_ONL	Graduate Online Enrollment	184.8	981.8	188.7	591.2
ONL_TOTAL	Total Online Enrollment	513.1	2,869.3	1,997.8	3,017.9
AV_SAL	Average Instructor Salary	75,415.8	47,584.3	75,376.9	47,476.9
TOTAL_EXP	Total Expenditures	98,944,322	418,303,090	158,362,359	440,699,670
<i>Observations:</i>		<i>1,866</i>		<i>1,917</i>	

*\*Source: Integrated Postsecondary Education Data System*

**Table 2: TransLog Cost Function***Total Online Classes*

Variable	Public Universities		Private Universities	
	Regression Coefficient	Standard Error	Regression Coefficient	Standard Error
Intercept	12.577**	0.54	13.657**	0.468
LN_UGRAD	0.376**	0.128	-0.427**	0.099
LN_GRAD	0.764**	0.193	0.041	0.112
LN_RES_TOTAL	-0.08	0.070	0.064	0.038
LN_ONL_TOTAL	0.405**	0.098	0.549**	0.104
MED_SCH	0.578**	0.063	0.426**	0.089
LN_UGRAD_2	0.038**	0.003	0.081**	0.003
LN_GRAD_2	0.030**	0.004	0.022**	0.003
LN_RES_TOTAL_2	0.009**	0.001	0.0129**	0.001
LN_ONL_TOTAL_2	0.017**	0.002	0.026**	0.002
LN_UGRAD_GRAD	-0.057**	0.005	-0.063**	0.003
LN_UGRAD_RES	-0.003	0.002	-0.005**	0.001
LN_UGRAD_ONL_TOTAL	-0.053**	0.003	-0.036**	0.002
LN_GRAD_RES	-0.005**	0.001	-0.005**	0.001
LN_GRAD_ONL_TOTAL	-0.003	0.003	-0.017**	0.002
LN_RES_ONL_TOTAL	-0.005**	0.001	0.001	0.001
LN_AV_SAL_INST	-0.580**	0.062	-0.458**	0.070
LN_AV_SAL_INST_2	0.045**	0.004	0.032**	0.004
LN_UGRAD_SAL_INST	-0.002	0.012	0.044**	0.009
LN_GRAD_SAL_INST	-0.028	0.018	0.032**	0.010
LN_RES_SAL_INST	0.004	0.006	-0.015**	0.003
LN_ONL_TOTAL_SAL_INST	-0.003	0.009	-0.033**	0.010
<i>N</i>	1914		1856	
<i>R</i> <sup>2</sup>	0.9514		0.9024	

\*Significant at the 5% level, two-tailed test

\*\*Significant at the 1% level, two-tailed test

**Table 3: TransLog Cost Function**  
*Online Graduate and Undergraduate Classes*

Variable	Public Universities		Private Universities	
	Regression Coefficient	Standard Error	Regression Coefficient	Standard Error
Intercept	11.968**	0.536	13.643**	0.459
LN_GRAD	1.372**	0.247	0.081	0.115
LN_UGRAD	0.493**	0.127	-0.350**	0.109
LN_RES_TOTAL	-0.081	0.069	0.042	0.037
LN_UGRAD_ONL	0.462**	0.098	0.347*	0.138
LN_GRAD_ONL	-0.957**	0.266	0.278	0.141
MED_SCH	0.548**	0.063	0.388**	0.086
LN_GRAD_2	0.042**	0.005	0.024**	0.003
LN_UGRAD_2	0.047**	0.003	0.090**	0.003
LN_RES_TOTAL_2	0.008**	0.001	0.013**	0.001
LN_UGRAD_ONL_2	0.022**	0.002	0.013**	0.003
LN_GRAD_ONL_2	0.013*	0.005	0.027**	0.004
LN_UGRAD_GRAD	-0.082**	0.006	-0.076**	0.003
LN_UGRAD_RES	-0.005*	0.002	-0.005**	0.001
LN_UGRAD_UGRAD_ONL	-0.064**	0.003	-0.048**	0.003
LN_UGRAD_GRAD_ONL	0.029**	0.005	-0.012**	0.003
LN_GRAD_RES	0.0004**	0.002	-0.003**	0.001
LN_GRAD_UGRAD_ONL	0.011**	0.004	0.006	0.003
LN_GRAD_GRAD_ONL	-0.026**	0.006	-0.019**	0.004
LN_RES_UGRAD_ONL	-0.004*	0.001	0.003*	0.001
LN_RES_GRAD_ONL	-0.006**	0.002	-0.002	0.001
LN_UGRAD_ONL_GRAD_ONL	-0.018**	0.006	-0.012**	0.004
LN_AV_SAL_INST	-0.550	0.061	-0.480**	0.068
LN_AV_SAL_INST_2	0.048**	0.003	0.033**	0.004
LN_UGRAD_SAL_INST	-0.01846	0.01176	0.03451**	0.01008
LN_GRAD_SAL_INST	-0.08045	0.02242	0.03204**	0.01070
LN_RES_SAL_INST	0.00527	0.00629	-0.01416**	0.00333
LN_SAL_INST_UGRAD_ONL	-0.00520	0.00904	-0.00723	0.01288
LN_SAL_INST_GRAD_ONL	0.08552**	0.02398	-0.01587	0.01292
<i>N</i>	<i>1914</i>		<i>1856</i>	
<i>R</i> <sup>2</sup>	<i>0.9531</i>		<i>0.9087</i>	

\*Significant at the 5% level, two-tailed test  
 \*\*Significant at the 1% level, two-tailed test

<b>Table 4: Economies of Scale and Scope</b>					
		<b>Scenario One</b>		<b>Scenario Two</b>	
		Public	Private	Public	Private
<b>Ray Economies of Scale</b>		<i>1.501</i>	<i>1.018</i>	<i>1.419</i>	<i>1.038</i>
<b>Economies of Scope</b>	Undergraduate Output	<i>2.617</i>	<i>0.556</i>	<i>7.120</i>	<i>3.763</i>
	Graduate Output	<i>0.151</i>	<i>0.971</i>	<i>4.643</i>	<i>0.613</i>
	Research Output	<i>-0.857</i>	<i>-0.402</i>	<i>-0.207</i>	<i>-0.141</i>
	Total Online Output	<i>0.684</i>	<i>0.797</i>	-	-
	Online Undergraduate Output	-	-	<i>0.846</i>	<i>-0.040</i>
	Online Graduate Output	-	-	<i>0.550</i>	<i>0.467</i>

*\*Scenarios 1-2 use results from Translog Cost Functions 1-2, respectively*

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## XI. SAS CODE

### (i) PRIVATE UNIVERSITY DATA

```
*** Honor's Senior Project: Private Universities ***
*** Author: Brandon Genetin ***
*** Data: Integrated Post Secondary Education Data System ***
*** Created: 2/23/2016 *** ;

data Private_1; ***Inputting Data;
infile '\\Client\E$\Senior Project\Senior_Project_Data_Private.csv'
delimiter=',' dsd missover firstobs=2 lrecl=32736;

ODS RTF FILE = '\\Client\E$\Senior Project\Private_Data.rtf';

informat
unitid 6.
instnm $50.
year 4.
MEDICAL 2.
EFUG 6.
EFGRAD 6.
F2E011 12.
F2E012 12.
F2E021 12.
F2E022 12.
F2E131 12.
PCUDEEXC 4.
PCUDESOM 4.
PCUDENON 4.
PCGDEEXC 4.
PCGDESOM 4.
PCGDENON 4.
SFTEPSTC 6.
SFTEINST 6.
SFTERSRC 6.;

input
unitid
instnm $
year
MEDICAL
EFUG
EFGRAD
F2E011
F2E012
F2E021
F2E022
F2E131
PCUDEEXC
PCUDESOM
PCUDENON
```

```
PCGDEEXC
PCGDESOM
PCGDENON
SFTEPSTC
SFTEINST
SFTERSRC ;
```

#### Label

```
unitid='Unique identification number for an institution'
instnm='Institution (entity) name'
MEDICAL='Institution grants a medical degree'
EFUG='Undergraduate enrollment'
EFGRAD='Graduate enrollment'
F2E011='Instruction-Total amount'
F2E012='Instruction-Salaries and wages'
F2E021='Research-Total amount'
F2E022='Research-Salaries and wages'
F2E131='Total expenses-Total amount'
PCUDEEXC='Percent of undergraduate students enrolled exclusively in
distance education courses'
PCUDESOM='Percent of undergraduate students enrolled in some but not
all distance education courses'
PCUDENON='Percent of undergraduate students not enrolled in any
distance education courses'
PCGDEEXC='Percent of graduate students enrolled exclusively in distance
education courses'
PCGDESOM='Percent of graduate students enrolled in some but not all
distance education courses'
PCGDENON='Percent of graduate students not enrolled in any distance
education courses'
SFTEPSTC='Instructional, research and public service FTE'
SFTEINST='Instructional FTE'
SFTERSRC='Research FTE';
```

```
proc format; ***Formatting Medical School Variable;
value MEDICAL 1='Yes'
2='No'
-1='Not reported'
-2='Not applicable';
```

```
data Private_2; ***Renaming Variables;
set Private_1;
rename unitid = ID;
rename instnm = NAME;
rename MEDICAL = MED_SCH;
rename F2E011 = INSTR_TOTAL;
rename F2E012 = INSTR_SALARY;
rename F2E021 = RES_TOTAL;
rename F2E022 = RES_SALARY;
rename F2E131 = TOTAL_EXP;
rename PCUDEEXC = UGRAD_ONLINE_EXC_PERCENT;
rename PCUDESOM = UGRAD_ONLINE_SOME_PERCENT;
rename PCUDENON = UGRAD_ONLINE_NONE_PERCENT;
```

```

rename PCGDEEXC = GRAD_ONLINE_EXC_PERCENT;
rename PCGDESOM = GRAD_ONLINE_SOME_PERCENT;
rename PCGDENON = GRAD_ONLINE_NONE_PERCENT;
rename SFTEPSTC = INSTR_RES_PUB_FTE;
rename SFTEINST = INSTR_FTE;
rename SFTERSRC = RESEARCH_FTE;
rename EFUG = UGRAD_FTE;
rename EFGRAD = GRAD_FTE;

run;

data Private_3; ***Deleting Not Reported or Not Specified Medical
Schools;
set Private_2;
  IF MED_SCH LT 0 THEN DELETE;
  ELSE IF MED_SCH = 2 THEN MED_SCH = 0;

run;

data Private_4; ***Creation of Salary Variables (Input Prices);
set Private_3;

  AV_SAL_INSTR = INSTR_SALARY/INSTR_FTE;

***Creation of Absolute Number of Students Taking Online Classes;
  if UGRAD_FTE GT 0 then UGRAD_ONLINE_EXC =
(UGRAD_ONLINE_EXC_PERCENT/100)*UGRAD_FTE;
  if UGRAD_FTE = 0 then UGRAD_ONLINE_EXC = 0;

  if UGRAD_FTE GT 0 then UGRAD_ONLINE_SOME =
(UGRAD_ONLINE_SOME_PERCENT/100)*UGRAD_FTE;
  if UGRAD_FTE = 0 then UGRAD_ONLINE_SOME = 0;

  if GRAD_FTE GT 0 then GRAD_ONLINE_EXC =
(GRAD_ONLINE_EXC_PERCENT/100)*GRAD_FTE;
  if GRAD_FTE = 0 then GRAD_ONLINE_EXC = 0;

  if GRAD_FTE GT 0 then GRAD_ONLINE_SOME =
(GRAD_ONLINE_SOME_PERCENT/100)*GRAD_FTE;
  if GRAD_FTE = 0 then GRAD_ONLINE_SOME = 0;

  if TOTAL_EXP = '.' then DELETE;

  if UGRAD_FTE = '.' then DELETE;

***Labeling of new variables;
Label
  AV_SAL_INSTR = 'Average Salary of Private Institution Instructor'
  UGRAD_ONLINE_EXC = 'Absolute Value of undergraduate students
enrolled exclusively in distance education courses'
  UGRAD_ONLINE_SOME = 'Absolute Value of undergraduate students
enrolled in some but not all distance education courses'
  GRAD_ONLINE_EXC = 'Absolute Value of graduate students enrolled
exclusively in distance education courses'

```

```
GRAD_ONLINE_SOME = 'Absolute Value of graduate students enrolled  
in some but not all distance education courses';
```

```
run;
```

```
data Private_5; ***Creation of Variables for Online Learning;  
set Private_4;
```

```
***Online Learning, Total;  
ONL_UGRAD_TOTAL = UGRAD_ONLINE_EXC + UGRAD_ONLINE_SOME;  
ONL_GRAD_TOTAL = GRAD_ONLINE_EXC + GRAD_ONLINE_SOME;
```

```
run;
```

```
data Private_6; ***Creation of Variable for Total Online Learning;  
set Private_5;
```

```
ONL_TOTAL = ONL_UGRAD_TOTAL + ONL_GRAD_TOTAL;  
UGRAD_FTE = UGRAD_FTE - ONL_UGRAD_TOTAL;  
GRAD_FTE = GRAD_FTE - ONL_GRAD_TOTAL;
```

```
IF GRAD_FTE < 0 then DELETE;
```

```
***Labeling of Online Variables;
```

```
Label
```

```
ONL_UGRAD_TOTAL = 'Total Number of Undergraduate Students Taking  
Online Classes'  
ONL_GRAD_TOTAL = 'Total Number of Graduate Students Taking Online  
Classes'  
ONL_TOTAL = 'Total Number of Students Taking Online Classes';
```

```
run;
```

```
data Private_7; ***Creation of Log Variables for Private;  
set Private_6;
```

```
LN_UGRAD = log(UGRAD_FTE+1);  
LN_GRAD = log(GRAD_FTE+1);  
LN_RES_TOTAL = log(RES_TOTAL+1);  
LN_ONL_TOTAL = log(ONL_TOTAL+1);  
LN_AV_SAL_INST = log(AV_SAL_INSTR+1);  
LN_TOTAL_EXP = log(TOTAL_EXP+1);  
LN_UGRAD_ONL = log(ONL_UGRAD_TOTAL+1);  
LN_GRAD_ONL = log(ONL_GRAD_TOTAL+1);
```

```
run;
```

```
data Private_8; ***Creation of Log Variables for Private, Squared;  
set Private_7;
```

```
LN_UGRAD_2 = LN_UGRAD**2;  
LN_GRAD_2 = LN_GRAD**2;  
LN_RES_TOTAL_2 = LN_RES_TOTAL**2;  
LN_ONL_TOTAL_2 = LN_ONL_TOTAL**2;  
LN_AV_SAL_INST_2 = LN_AV_SAL_INST**2;
```

```

LN_UGRAD_ONL_2 = LN_UGRAD_ONL**2;
LN_GRAD_ONL_2 = LN_GRAD_ONL**2;

run;

data Private_9; ***Creation of Interaction Terms for Private;
set Private_8;

***Interaction Terms for Undergraduate Enrollment;
LN_UGRAD_GRAD = LN_UGRAD * LN_GRAD;
LN_UGRAD_RES = LN_UGRAD * LN_RES_TOTAL;
LN_UGRAD_ONL_TOTAL = LN_UGRAD * LN_ONL_TOTAL;
LN_UGRAD_SAL_INST = LN_UGRAD * LN_AV_SAL_INST;

LN_UGRAD_UGRAD_ONL = LN_UGRAD * LN_UGRAD_ONL;
LN_UGRAD_GRAD_ONL = LN_UGRAD * LN_GRAD_ONL;

***Interaction Terms for Graduate Enrollment;
LN_GRAD_RES = LN_GRAD * LN_RES_TOTAL;
LN_GRAD_ONL_TOTAL = LN_GRAD * LN_ONL_TOTAL;
LN_GRAD_SAL_INST = LN_GRAD * LN_AV_SAL_INST;

LN_GRAD_UGRAD_ONL = LN_GRAD * LN_UGRAD_ONL;
LN_GRAD_GRAD_ONL = LN_GRAD * LN_GRAD_ONL;

***Interaction Terms for Research Expenditures, Private;
LN_RES_ONL_TOTAL = LN_RES_TOTAL * LN_ONL_TOTAL;
LN_RES_SAL_INST = LN_RES_TOTAL * LN_AV_SAL_INST;

LN_RES_UGRAD_ONL = LN_RES_TOTAL * LN_UGRAD_ONL;
LN_RES_GRAD_ONL = LN_RES_TOTAL * LN_GRAD_ONL;

***Interaction Terms for Online Classes;
LN_ONL_TOTAL_SAL_INST = LN_ONL_TOTAL * LN_AV_SAL_INST;

***Interaction Terms for Salary Instruction;
LN_SAL_INST_UGRAD_ONL = LN_AV_SAL_INST * LN_UGRAD_ONL;
LN_SAL_INST_GRAD_ONL = LN_AV_SAL_INST * LN_GRAD_ONL;

***Interaction Terms for Undergraduate Online;
LN_UGRAD_ONL_GRAD_ONL = LN_UGRAD_ONL * LN_GRAD_ONL;

run;

***Descriptive Statistics on Data Points of Interest;
proc means
data = Private_9;
run;

proc reg; ***Transcendental Logarithmic Function of Private University;
model LN_TOTAL_EXP = LN_UGRAD LN_GRAD LN_RES_TOTAL
LN_ONL_TOTAL MED_SCH LN_UGRAD_2 LN_GRAD_2
LN_RES_TOTAL_2 LN_ONL_TOTAL_2 LN_UGRAD_GRAD
LN_UGRAD_RES LN_UGRAD_ONL_TOTAL LN_GRAD_RES
LN_GRAD_ONL_TOTAL LN_RES_ONL_TOTAL;

```

```

run;

proc reg; ***Transcendental Logarithmic Function of Private University
with Input Prices;
model LN_TOTAL_EXP = LN_UGRAD LN_GRAD LN_RES_TOTAL
LN_ONL_TOTAL MED_SCH LN_UGRAD_2 LN_GRAD_2
LN_RES_TOTAL_2 LN_ONL_TOTAL_2 LN_UGRAD_GRAD
LN_UGRAD_RES LN_UGRAD_ONL_TOTAL LN_GRAD_RES
LN_GRAD_ONL_TOTAL LN_RES_ONL_TOTAL LN_AV_SAL_INST
LN_AV_SAL_INST_2 LN_UGRAD_SAL_INST LN_GRAD_SAL_INST
LN_RES_SAL_INST LN_ONL_TOTAL_SAL_INST;
run;

proc reg; ***Transcendental Logarithmic Function of Private University
with Graduate and Undergraduate Online Classes;
model LN_TOTAL_EXP = LN_GRAD LN_UGRAD
LN_RES_TOTAL LN_UGRAD_ONL LN_GRAD_ONL MED_SCH
LN_GRAD_2 LN_UGRAD_2 LN_RES_TOTAL_2 LN_UGRAD_ONL_2
LN_GRAD_ONL_2 LN_UGRAD_GRAD LN_UGRAD_RES
LN_UGRAD_UGRAD_ONL LN_UGRAD_GRAD_ONL LN_GRAD_RES
LN_GRAD_UGRAD_ONL LN_GRAD_GRAD_ONL LN_RES_UGRAD_ONL
LN_RES_GRAD_ONL LN_UGRAD_ONL_GRAD_ONL;
run;

proc reg; ***Transcendental Logarithmic Function of Private University
with Graduate and Undergraduate Online Classes and Input Prices;
model LN_TOTAL_EXP = LN_GRAD LN_UGRAD LN_RES_TOTAL
LN_UGRAD_ONL LN_GRAD_ONL MED_SCH LN_GRAD_2
LN_UGRAD_2 LN_RES_TOTAL_2 LN_UGRAD_ONL_2
LN_GRAD_ONL_2 LN_UGRAD_GRAD LN_UGRAD_RES
LN_UGRAD_UGRAD_ONL LN_UGRAD_GRAD_ONL LN_GRAD_RES
LN_GRAD_UGRAD_ONL LN_GRAD_GRAD_ONL LN_RES_UGRAD_ONL
LN_RES_GRAD_ONL LN_UGRAD_ONL_GRAD_ONL LN_AV_SAL_INST
LN_AV_SAL_INST_2 LN_UGRAD_SAL_INST LN_GRAD_SAL_INST
LN_RES_SAL_INST LN_SAL_INST_UGRAD_ONL
LN_SAL_INST_GRAD_ONL;
run;

ODS RTF CLOSE;

```

## (ii) PUBLIC UNIVERSITY DATA

```

*** Honor's Senior Project: Public Universities ***
*** Author: Brandon Genetin ***
*** Data: Integrated Post Secondary Education Data System ***
*** Created: 2/23/2016 *** ;

data Public_1; ***Inputting Data;
infile '\\Client\E$\Senior Project\Senior_Project_Data_Public.csv'
delimiter=', ' dsd missover firstobs=2 lrecl=32736;

ODS RTF FILE = '\\Client\E$\Senior Project\Public_Data.rtf';

informat

```

```
unitid 6.  
instnm $50.  
year 4.  
MEDICAL 2.  
EFUG 6.  
EFGRAD 6.  
F1C011 12.  
F1C012 12.  
F1C021 12.  
F1C022 12.  
F1C191 12.  
PCUDEEXC 4.  
PCUDESOM 4.  
PCUDENON 4.  
PCGDEEXC 4.  
PCGDESOM 4.  
PCGDENON 4.  
SFTEPSTC 6.  
SFTEINST 6.  
SFTERSRC 6.;
```

```
input  
unitid  
instnm $  
year  
MEDICAL  
EFUG  
EFGRAD  
F1C011  
F1C012  
F1C021  
F1C022  
F1C191  
PCUDEEXC  
PCUDESOM  
PCUDENON  
PCGDEEXC  
PCGDESOM  
PCGDENON  
SFTEPSTC  
SFTEINST  
SFTERSRC ;
```

```
Label  
unitid='Unique identification number for an institution'  
instnm='Institution (entity) name'  
MEDICAL='Institution grants a medical degree'  
EFUG='Undergraduate enrollment'  
EFGRAD='Graduate enrollment'  
F1C011='Instruction - Current year total'  
F1C012='Instruction - Salaries and wages'  
F1C021='Research - Current year total'  
F1C022='Research - Salaries and wages'
```

```

F1C191='Total expenses deductions - Current year total'
PCUDEEXC='Percent of undergraduate students enrolled exclusively in
distance education courses'
PCUDESOM='Percent of undergraduate students enrolled in some but not
all distance education courses'
PCUDENON='Percent of undergraduate students not enrolled in any
distance education courses'
PCGDEEXC='Percent of graduate students enrolled exclusively in distance
education courses'
PCGDESOM='Percent of graduate students enrolled in some but not all
distance education courses'
PCGDENON='Percent of graduate students not enrolled in any distance
education courses'
SFTEPSTC='Instructional, research and public service FTE'
SFTEINST='Instructional FTE'
SFTERSRC='Research FTE';

```

```

proc format; ***Formatting Medical School Variable;
value MEDICAL 1='Yes'
2='No'
-1='Not reported'
-2='Not applicable';

```

```

data Public_2; ***Renaming Variables;
  set Public_1;
  rename unitid = ID;
  rename instnm = NAME;
  rename MEDICAL = MED_SCH;
  rename F1C011 = INSTR_TOTAL;
  rename F1C012 = INSTR_SALARY;
  rename F1C021 = RES_TOTAL;
  rename F1C022 = RES_SALARY;
  rename F1C191 = TOTAL_EXP;
  rename PCUDEEXC = UGRAD_ONLINE_EXC_PERCENT;
  rename PCUDESOM = UGRAD_ONLINE_SOME_PERCENT;
  rename PCUDENON = UGRAD_ONLINE_NONE_PERCENT;
  rename PCGDEEXC = GRAD_ONLINE_EXC_PERCENT;
  rename PCGDESOM = GRAD_ONLINE_SOME_PERCENT;
  rename PCGDENON = GRAD_ONLINE_NONE_PERCENT;
  rename SFTEPSTC = INSTR_RES_PUB_FTE;
  rename SFTEINST = INSTR_FTE;
  rename SFTERSRC = RESEARCH_FTE;
  rename EFUG = UGRAD_FTE;
  rename EFGRAD = GRAD_FTE;

```

```

run;

```

```

data Public_3; ***Deleting Not Reported or Not Specified Medical
Schools;
set Public_2;
  IF MED_SCH LT 0 THEN DELETE;
  ELSE IF MED_SCH = 2 THEN MED_SCH = 0;

```

```

run;

```

```

data Public_4; ***Creation of Salary Variables (Input Prices);
set Public_3;

    AV_SAL_INSTR = INSTR_SALARY/INSTR_FTE;

***Creation of Absolute Number of Students Taking Online Classes;
    if UGRAD_FTE GT 0 then UGRAD_ONLINE_EXC =
(UGRAD_ONLINE_EXC_PERCENT/100)*UGRAD_FTE;
    if UGRAD_FTE = 0 then UGRAD_ONLINE_EXC = 0;

    if UGRAD_FTE GT 0 then UGRAD_ONLINE_SOME =
(UGRAD_ONLINE_SOME_PERCENT/100)*UGRAD_FTE;
    if UGRAD_FTE = 0 then UGRAD_ONLINE_SOME = 0;

    if GRAD_FTE GT 0 then GRAD_ONLINE_EXC =
(GRAD_ONLINE_EXC_PERCENT/100)*GRAD_FTE;
    if GRAD_FTE = 0 then GRAD_ONLINE_EXC = 0;

    if GRAD_FTE GT 0 then GRAD_ONLINE_SOME =
(GRAD_ONLINE_SOME_PERCENT/100)*GRAD_FTE;
    if GRAD_FTE = 0 then GRAD_ONLINE_SOME = 0;

    if TOTAL_EXP = '.' then DELETE;

    if UGRAD_FTE = '.' then DELETE;

***Labeling of new variables;
Label
    AV_SAL_INSTR = 'Average Salary of Private Institution Instructor'
    UGRAD_ONLINE_EXC = 'Absolute Value of undergraduate students
enrolled exclusively in distance education courses'
    UGRAD_ONLINE_SOME = 'Absolute Value of undergraduate students
enrolled in some but not all distance education courses'
    GRAD_ONLINE_EXC = 'Absolute Value of graduate students enrolled
exclusively in distance education courses'
    GRAD_ONLINE_SOME = 'Absolute Value of graduate students enrolled
in some but not all distance education courses';

run;

data Public_5; ***Creation of Variables for Online Learning;
set Public_4;

    ***Online Learning, Total;
    ONL_UGRAD_TOTAL = UGRAD_ONLINE_EXC + UGRAD_ONLINE_SOME;
    ONL_GRAD_TOTAL = GRAD_ONLINE_EXC + GRAD_ONLINE_SOME;

run;

data Public_6; ***Creation of Variable for Total Online Learning;
set Public_5;

    ONL_TOTAL = ONL_UGRAD_TOTAL + ONL_GRAD_TOTAL;

```

```

UGRAD_FTE = UGRAD_FTE - ONL_UGRAD_TOTAL;
GRAD_FTE = GRAD_FTE - ONL_GRAD_TOTAL;

IF GRAD_FTE < 0 then DELETE;

***Labeling of Online Variables;
Label
ONL_UGRAD_TOTAL = 'Total Number of Undergraduate Students Taking
Online Classes'
ONL_GRAD_TOTAL = 'Total Number of Graduate Students Taking Online
Classes'
ONL_TOTAL = 'Total Number of Students Taking Online Classes';

run;

data Public_7; ***Creation of Log Variables for Public;
set Public_6;
LN_UGRAD = log(UGRAD_FTE+1);
LN_GRAD = log(GRAD_FTE+1);
LN_RES_TOTAL = log(RES_TOTAL+1);
LN_ONL_TOTAL = log(ONL_TOTAL+1);
LN_AV_SAL_INST = log(AV_SAL_INSTR+1);
LN_TOTAL_EXP = log(TOTAL_EXP+1);
LN_UGRAD_ONL = log(ONL_UGRAD_TOTAL+1);
LN_GRAD_ONL = log(ONL_GRAD_TOTAL+1);

run;

data Public_8; ***Creation of Log Variables for Public, Squared;
set Public_7;
LN_UGRAD_2 = LN_UGRAD**2;
LN_GRAD_2 = LN_GRAD**2;
LN_RES_TOTAL_2 = LN_RES_TOTAL**2;
LN_ONL_TOTAL_2 = LN_ONL_TOTAL**2;
LN_AV_SAL_INST_2 = LN_AV_SAL_INST**2;

LN_UGRAD_ONL_2 = LN_UGRAD_ONL**2;
LN_GRAD_ONL_2 = LN_GRAD_ONL**2;

run;

data Public_9; ***Creation of Interaction Terms for Public;
set Public_8;

***Interaction Terms for Undergraduate Enrollment;
LN_UGRAD_GRAD = LN_UGRAD * LN_GRAD;
LN_UGRAD_RES = LN_UGRAD * LN_RES_TOTAL;
LN_UGRAD_ONL_TOTAL = LN_UGRAD * LN_ONL_TOTAL;
LN_UGRAD_SAL_INST = LN_UGRAD * LN_AV_SAL_INST;

LN_UGRAD_UGRAD_ONL = LN_UGRAD * LN_UGRAD_ONL;
LN_UGRAD_GRAD_ONL = LN_UGRAD * LN_GRAD_ONL;

***Interaction Terms for Graduate Enrollment;
LN_GRAD_RES = LN_GRAD * LN_RES_TOTAL;

```

```

LN_GRAD_ONL_TOTAL = LN_GRAD * LN_ONL_TOTAL;
LN_GRAD_SAL_INST = LN_GRAD * LN_AV_SAL_INST;

LN_GRAD_UGRAD_ONL = LN_GRAD * LN_UGRAD_ONL;
LN_GRAD_GRAD_ONL = LN_GRAD * LN_GRAD_ONL;

***Interaction Terms for Research Expenditures, Public;
LN_RES_ONL_TOTAL = LN_RES_TOTAL * LN_ONL_TOTAL;
LN_RES_SAL_INST = LN_RES_TOTAL * LN_AV_SAL_INST;

LN_RES_UGRAD_ONL = LN_RES_TOTAL * LN_UGRAD_ONL;
LN_RES_GRAD_ONL = LN_RES_TOTAL * LN_GRAD_ONL;

***Interaction Terms for Online Classes;
LN_ONL_TOTAL_SAL_INST = LN_ONL_TOTAL * LN_AV_SAL_INST;

***Interaction Terms for Salary Instruction;
LN_SAL_INST_UGRAD_ONL = LN_AV_SAL_INST * LN_UGRAD_ONL;
LN_SAL_INST_GRAD_ONL = LN_AV_SAL_INST * LN_GRAD_ONL;

***Interaction Terms for Undergraduate Online;
LN_UGRAD_ONL_GRAD_ONL = LN_UGRAD_ONL * LN_GRAD_ONL;

run;

***Descriptive Statistics on Data Points of Interest;
proc means
    data = Public_9;
run;

proc reg; ***Transcendental Logarithmic Function of Public University;
model LN_TOTAL_EXP = LN_UGRAD LN_GRAD LN_RES_TOTAL
LN_ONL_TOTAL MED_SCH LN_UGRAD_2 LN_GRAD_2
LN_RES_TOTAL_2 LN_ONL_TOTAL_2 LN_UGRAD_GRAD
LN_UGRAD_RES LN_UGRAD_ONL_TOTAL LN_GRAD_RES
LN_GRAD_ONL_TOTAL LN_RES_ONL_TOTAL;
run;

proc reg; ***Transcendental Logarithmic Function of Public University
with Input Prices;
model LN_TOTAL_EXP = LN_UGRAD LN_GRAD LN_RES_TOTAL
LN_ONL_TOTAL MED_SCH LN_UGRAD_2 LN_GRAD_2
LN_RES_TOTAL_2 LN_ONL_TOTAL_2 LN_UGRAD_GRAD
LN_UGRAD_RES LN_UGRAD_ONL_TOTAL LN_GRAD_RES
LN_GRAD_ONL_TOTAL LN_RES_ONL_TOTAL LN_AV_SAL_INST
LN_AV_SAL_INST_2 LN_UGRAD_SAL_INST LN_GRAD_SAL_INST
LN_RES_SAL_INST LN_ONL_TOTAL_SAL_INST;
run;

proc reg; ***Transcendental Logarithmic Function of Public University
with Graduate and Undergraduate Online Classes;
model LN_TOTAL_EXP = LN_GRAD LN_UGRAD LN_RES_TOTAL
LN_UGRAD_ONL LN_GRAD_ONL MED_SCH LN_GRAD_2
LN_UGRAD_2 LN_RES_TOTAL_2 LN_UGRAD_ONL_2
LN_GRAD_ONL_2 LN_UGRAD_GRAD LN_UGRAD_RES
LN_UGRAD_UGRAD_ONL LN_UGRAD_GRAD_ONL

```

```

LN_GRAD_RES                LN_GRAD_UGRAD_ONL                LN_GRAD_GRAD_ONL
LN_RES_UGRAD_ONL          LN_RES_GRAD_ONL          LN_UGRAD_ONL_GRAD_ONL;
run;

proc reg; ***Transcendental Logarithmic Function of Public University
with Graduate and Undergraduate Online Classes and Input Prices;
model LN_TOTAL_EXP =      LN_GRAD                LN_UGRAD
LN_RES_TOTAL            LN_UGRAD_ONL            LN_GRAD_ONL
MED_SCH                LN_GRAD_2                LN_UGRAD_2                LN_RES_TOTAL_2
LN_UGRAD_ONL_2          LN_GRAD_ONL_2          LN_UGRAD_GRAD
LN_UGRAD_RES            LN_UGRAD_UGRAD_ONL            LN_UGRAD_GRAD_ONL
LN_GRAD_RES            LN_GRAD_UGRAD_ONL            LN_GRAD_GRAD_ONL
LN_RES_UGRAD_ONL        LN_RES_GRAD_ONL            LN_UGRAD_ONL_GRAD_ONL
LN_AV_SAL_INST          LN_AV_SAL_INST_2        LN_UGRAD_SAL_INST
LN_GRAD_SAL_INST        LN_RES_SAL_INST            LN_SAL_INST_UGRAD_ONL
LN_SAL_INST_GRAD_ONL;
run;

ODS RTF CLOSE

```